Solutions:

1. State from the tables the value of Gibbs free energy for water vapour at 300K. In calculating the Equilibrium constant for reactions with water, what further consideration is required for the Gibbs free energy? [3 marks]

1 Gibbs free energy of water vapour, from p. 18 of tables at 300K is −56 616 kJ/kmol.
For equilibrium constant calculations with H_2O, it is necessary to include the formation enthalpy of H_2O since the state ~~enthalpy~~ Gibbs free energy does not include it.

2. Calculate the specific entropy of carbon dioxide at 600K and 20 bar.

2 CO_2 at 600K : $s = 243.2$ kJ/kmol K

Using the formula: $s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

& $T_2 = T_1$

$$s_2 - \frac{243.2\ (kJ/kmol K)}{44\ (kg/kmol)} = -\frac{8.314}{44} \ln 20$$

$$\rightarrow s_2 = -0.566 + 5.523$$

$$= 4.957 \text{ kJ/kg K}.$$

[3 marks]

3. Briefly describe three implications of using organic Rankine cycles.

4. Any three of the following: ORC works generally at lower temperatures, extracting heat to produce work. Organic compounds degrade at high temperatures. ~~toxic & flammable~~ - risks. Vacuum can be avoided. Low pressure system. Compact. Often positive isentropic saturated vapour curve – no wet turbine exhaust

[3 marks]

4. Use the non-steady flow energy equation to derive an expression for the flow of superheated steam into an initially evacuated closed vessel. [3 marks]

d NSFEE:

$$\dot{Q} - \dot{W} + p\frac{dV}{dt} = \sum \dot{m}_o\left(h_o + \frac{C_o^2}{2} + g z_o\right) + \sum \dot{m}_i\left(h_i + \frac{C_i^2}{2} + g z_i\right) + \frac{d}{dt} m_{cv}\left(u_{cv} + \frac{C_{cv}^2}{2} + g z_{cv}\right)$$

assume negligible ke & pe

$$\therefore \sum \dot{m}_i h_i = \frac{d}{dt} m_{cv} u_{cv}$$

$$\int \dot{m}_i h_i \, dt = \int u_{cv} \, dm_{cv} \Rightarrow h_i m_{cv} = m_{cv} u_t \Rightarrow h_i = u_t$$

5. State two advantages of an Integrated Coal Gasification Combined Cycle Power Plant over a conventional coal-fired power plant. [3 marks]

With IGCC, CO_2 can be captured more easily, and the syngas can be used for CCGT.

6. For radiation, define and explain the concept of the view factor and a diffuse surface [4]

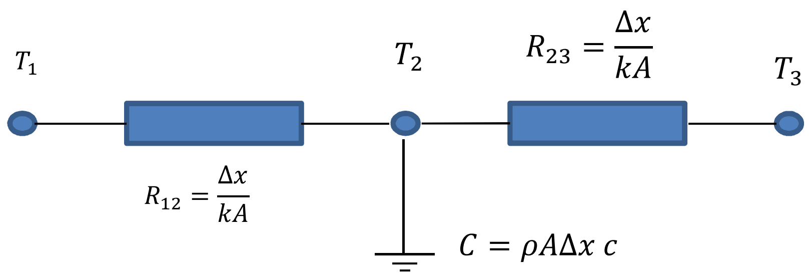
View factor is the proportion of energy (relative to 1-all) that is emitted from one surface that is seen by a second surface. (2 marks). A diffuse surface is a surface which emits energy in all directions equally. (2 marks)

7. In my shower room, the water vapour in the air condenses on the cold mirror. If I wished to model the system as a mass transfer problem, what boundary conditions for the humidity should I set at the surface of the mirror? [3]

There is 100% humidity on the surface (covered in water) (1 mark) and a sudden jump to lower humidity near the surface (2 mark)

8. Fig Q5 shows the resistance network for internal nodes in an unsteady problem. Eq. Q5 shows the balance of energy in this resistance network.

Fig. Q5



$$\frac{kA}{\Delta x}(T_1^0 - T_2^0) + \frac{kA}{\Delta x}(T_3^0 - T_2^0) = \rho A \Delta x c \frac{T_2^1 - T_2^0}{dt} \quad (Q5)$$

(a) Are we using the explicit or implicit solution of the differential equation? [2]

Explicit solution because temperature are from the initial step.

(b) Rearrange the equation in terms of the Fourier number and collect each temperature term. Show working. Marks are given for clarity of presentation. [3]

$$\frac{kA}{\Delta x}(T_1^0 - T_2^0) + \frac{kA}{\Delta x}(T_3^0 - T_2^0) = \rho A \Delta x c \frac{T_2^1 - T_2^0}{dt}$$

$$(T_1^0 - T_2^0) + (T_3^0 - T_2^0) = \left(\frac{\rho c}{k}\right)\left(\frac{A}{A}\right)\left(\frac{\Delta x^2}{dt}\right)(T_2^1 - T_2^0)$$

$$(T_1^0 - T_2^0) + (T_3^0 - T_2^0) = \left(\frac{\Delta x^2}{\alpha\, dt}\right)(T_2^1 - T_2^0)$$

$$T_2^1 = T_2^0 + Fo(T_1^0 - T_2^0) + Fo(T_3^0 - T_2^0) \quad (1\ mark\ for\ changing\ to\ Fo)$$

$$T_2^1 = T_2^0(1 - 2Fo) + FoT_1^0 + FoT_3^0 \ (1\ mark\ for\ collecting\ to\ temperatures)$$

$$1\ mark\ for\ clear\ steps$$

(c) What is the criteria for stability for this internal node? [2]

$$(1-2Fo) \geq 0, or\ Fo \leq 0.5$$

9. Biot number and Nusselt number are both in the form $\frac{hL}{k}$. What are their physical significance and emphasise the differences between these in terms of the variables employed? [4]

Biot number is $Bi = \frac{h\Delta x}{k_{solid}}$, Ratio of thermal resistance through a solid to thermal resistance into the solid

Nusselt number is $Nu = \frac{hL}{k_{air}}$, Ratio of heat loss due to convection compared to the heat loss that would occur form the surface if there was only conduction.

Therefore, the values of thermal conductivity are different. In Biot it is for the solid, while in the Nusselt number the thermal conductivity is for the air.

10. A vertically mounted panel on a wall is electrically heated is used to keep a room at constant temperature. The room is kept at 20^oC, and the panel is heated to a constant 84^oC. The panel has dimensions 1 meter in the horizontal direction and 0.5 m in the vertical direction. Calculate the Rayleigh number for the flow and state if the flow is laminar ($Ra_L < 10^{13}$) or turbulent ($Ra_L > 10^{13}$). [8]

$$Ra_L = \frac{g\beta(T_p - T_\infty)L^3}{\alpha\nu}, \alpha = \frac{k}{\rho c}$$

Film temperature is $T_f = \frac{T_p + T_{air}}{2} = \frac{84+20}{2} = 52^oC = 325\ K$ (1 mark)

At this temperature the values of the variables are (4 marks):

$$\beta = \frac{1}{325}\ K^{-1}, k = 2.816 \times 10^{-2} W\ m^{-1}K^{-1}\quad \rho = 1.086\ kgm^{-3}, c = 1006.3\ J\ kg^{-1}K^{-1},$$

$$\nu = 1.807 \times 10^{-5} m^2 s^{-1}, L = 0.5\ m,$$

$$(\ 1\ marks\ lose\ 0.5\ mark\ for\ incorrect\ units\ )$$

The panel is mounted vertically so the characteristic length is wrong, the horizontal dimension is wrong so use the vertical dimension. Students that used the wrong length get maximum of 5/8,

This gives a value of the Rayleigh number of$Ra_L = 5.18E8$. (1 mark) This is laminar (1 mark)

11. For what conditions is the Reynolds analogy valid? [2]

The Reynolds analogy assumes that the mechanisms for heat transport are the same for momentum transport i.e. the mixing is cause by inertial forces. (1 mark) It is therefore valid if the ratio of the thermal and momentum diffusion values are about 1 (Prandtl number ~1) (1 mark)

12. What is the maximum size of computational cells you can use if you are numerically modelling transient heat transfer for the case of convection into a surface with thermal conductivity k=432W/mK and a heat transfer coefficient of h=50 W/m^{2}K and wish a stable solution? [3]

$$Bi = \frac{h\Delta x}{k} \rightarrow \Delta x = Bi\frac{k}{h}$$

(1 mark) For accurate solutions we need to use $Bi < 1, or\ Bi = 0.1$. (1 mark)
So $\Delta x < 0.864\ m$ (1 mark)

13. What is thermal contact resistance and explain how the use of "thermal greases, pastes or gels" improves the performance over those of air. Use a diagram. [4]

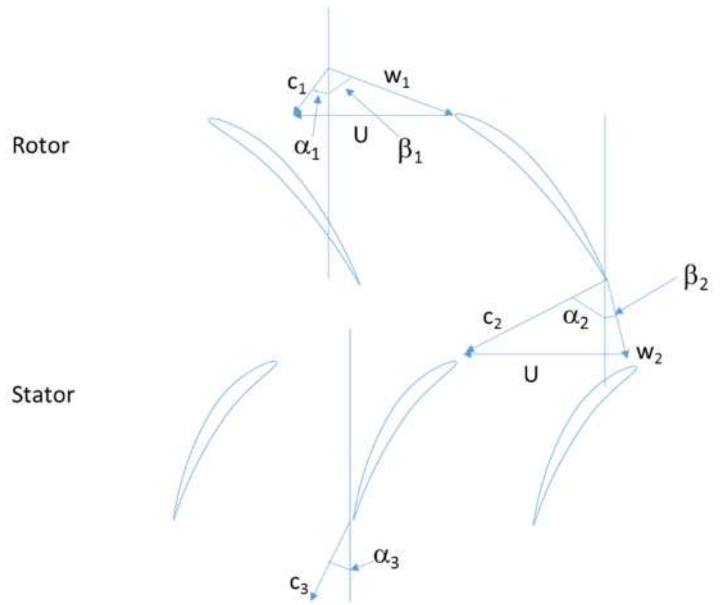
Contact resistance is what happens because the join of a material contains gaps. (2 mark if informative diagram) These gaps (usually filled with air) will act as a barrier to the transportation of heat and can be categorised by the use of a thermal resistance value called the contact resistance.(1 mark) Thermal greases etc will fill the gap and offer an easier path for the heat to travel over the gap(1 mark).

14. Briefly explain, with reference to the Euler work equation, $\dot{W}_c = \dot{m}(U_2 c_{\theta 2} - U_1 c_{\theta 1})$, why the absolute exit flow angle determines work on a rotor. [4 marks]

From the Euler work equation it is seen that there is a contribution of the absolute tangential velocity, c_θ, and of the tangential velocity of the blade. The absolute tangential exit velocity therefore has a significant effect on the work produced, which is controlled by the flow rate and the aerofoil geometry.

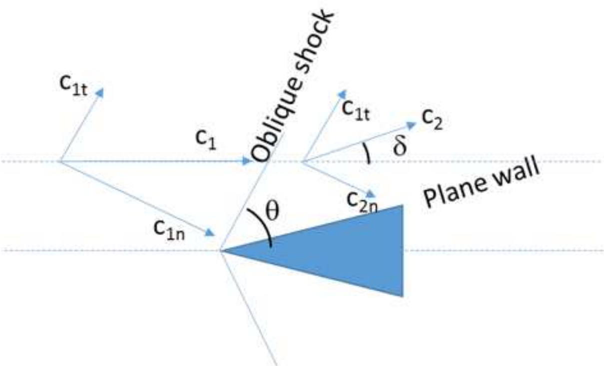
15. Sketch the typical blade configuration and velocity triangles involved in an axial compressor stage. [3 marks]

All that is required is the correct sketch, but here are also some helpful pointers to how to think it through. From the notes, it can be recalled that the rotor precedes the stator in the compressor, and the important things to notice are the velocity triangles between each part – at entry there is the blade velocity and the angle of attack which define the direction of the absolute velocity entering. After the rotor, there is the same blade velocity in an axial compressor, and the approximate relative velocity defined by blade angle defining the absolute exit velocity. The exit from the stator is defined by the blade exit angle and provides the entry to the next stage – which for a normal compressor stage, in which the entry directions are the same in each stage is the same as the first entry velocity.



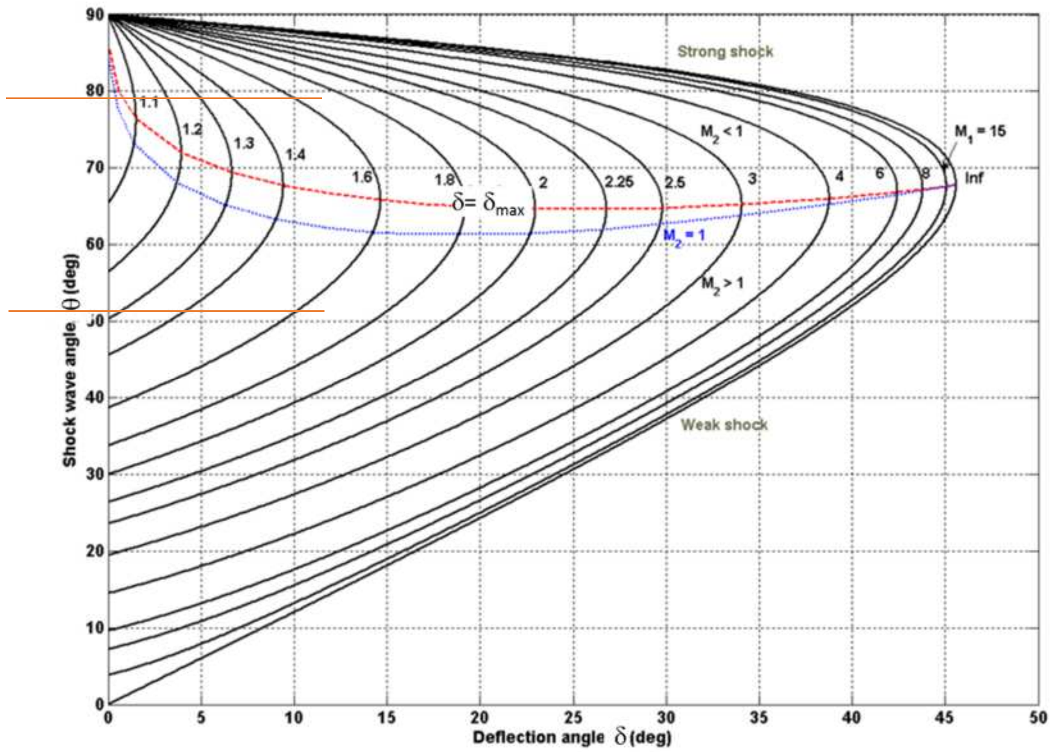
16. Sketch the situation of an oblique shock indicating the key angles involved. Using the chart in Figure Q16, determine the possible shock wave angles for a 10° wall at Mach 1.6. [6 marks]

The sketch can be recalled from the notes. The important thing to notice is the resolving parallel and perpendicular to the shock wave (assume a shock wave angle) and that the parallel to shock velocity does not change, but that the perpendicular is affected in the same way as a normal shock for that component.



[3]

Using the chart: for 10°, and for M = 1.6, the angles are approximately 79° strong shock, and the more likely approximately 52° weak shock. Approximate identification of angles 2 marks and stating strong and weak 1 mark.



Strong shock
Weak shock
$M_2 < 1$
$M_2 = 1$
$M_2 > 1$
$M_1 = 15$
Inf
$\delta = \delta_{max}$
1.1
1.2
1.3
1.4
1.6
1.8
2
2.25
2.5
3
4
6
8
Shock wave angle θ (deg)
Deflection angle δ (deg)

[3]

Part B:

17. Figure QB1 shows a schematic diagram, at a particular instant of the engine cycle, of a cylinder head (Surface 1), piston crown (Surface 2) and cylinder liner (Surface 3).

(a) Using the dimensions indicated on the diagram, and given that $F_{12} = 0.6$, calculate all view factors. [8]

Need areas so

$$A_1 = \pi * 0.05^2 = 0.00785\ m^2; A_2 = 0.00785; A_3 = 0.025 * \pi 0.1 = 0.00785 m^2$$

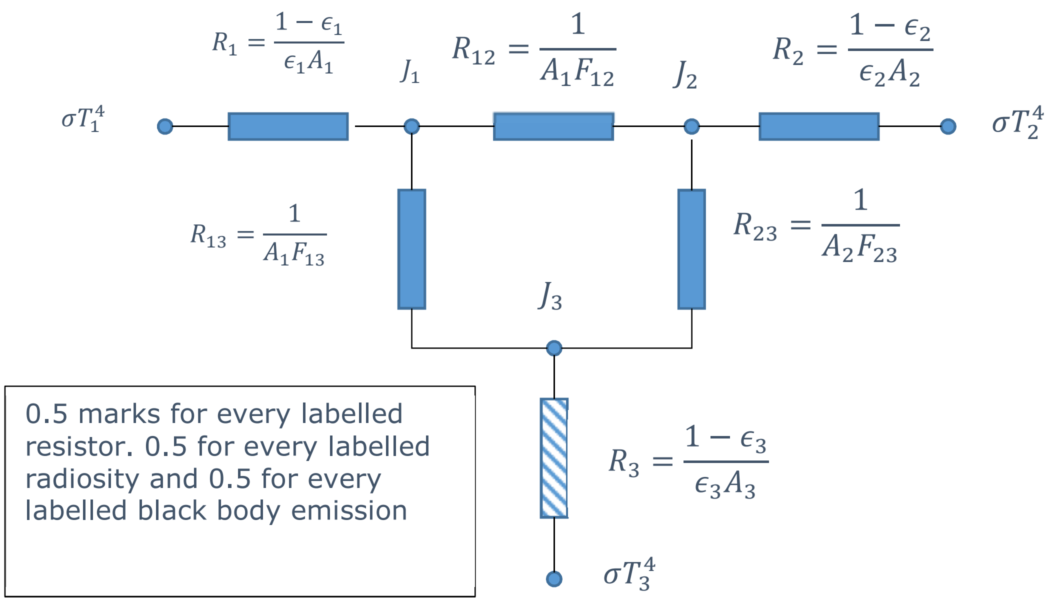
$$F_{11} = 0, F_{12} = 0.6(given), F_{13} = 1 - F_{12} = 0.4$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = 0.6, F_{22} = 0, F_{23} = 1 - F_{21} = 0.4$$

$$F_{31} = \frac{A_1}{A_3} F_{13} = \mathbf{0.4}, F_{32} = \frac{A_2}{A_3} F_{23} = 0.4, F_{33} = 1 - F_{31} - F_{32} = 0.2$$

1 mark for each correct view factor.

(b) Draw the resistance network for this problem. Identify all temperatures, and resistances on the diagram [6]



(c) The cylinder head can be represented as a **black body** at the temperature $T_1 = 1700K$, the cylinder lining is well insulated so that it is **adiabatic** and the emissivity of the piston is $\epsilon_2 = 0.75$. What is the radiosity of the cylinder head using this information and why? [3]

For a black body, the radiosity is equal to the black body emission (2 mark).

So heat flow through R_1 is zero. Mathematically it can be written as $\frac{J_1 - \sigma T_1^4}{R_1} = 0$, multiply both sides by R_1 to get $J_1 - \sigma T_1^4 = 0$.

In other words:

$$J_1 = \sigma T_1^4 = 5.67E - 8 * (1700)^4 = 4.735 \times 10^5\ W m^{-2}\ (1\ mark)$$

The radiosity is not dependent on the area here, it has units W/m^2. The heat flow **from** J_1 is the Radiosity times the area. Look at units.

(d) If the surface temperature of the piston crown is, $T_2 = 600\ K$, and using the conservation of heat flow, calculate the radiosity for the piston crown. [4]

From energy conservation, heat entering at surface 1 is equal to heat leaving at surface 2(2 mark). So the heat flow through R_1 is equal to the heat for through R_2 and the heat through the entire system. So we need to estimate the resistance network and apply it to get the heat flow first. First step is to get the parallel resistance RD:

$$R_D = \left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{32}}\right)^{-1} = 159\ K/W$$

The total resistance is then

$$R_T = R_D + R_2 = 159 + 42 = 201.6\ K/W \quad (1\ mark)$$

This gives a heat flow of:

$$Q = \frac{(\sigma T_1^4 - \sigma T_2^4)}{RT} = 2313\ W \quad (1\ mark)$$

We now know that the heat flow Q through R2.

$$Q = \frac{J_2 - \sigma T_1^4}{R2} \rightarrow J_2 = \sigma T_2^4 + R2\ Q = 10550 W/m^2 \quad (1\ mark)$$

(e) Using the radiosity of the head and piston, calculate the temperature of the cylinder lining. [6]

This is simply the nodal analysis of the node for J_3 and then the recognition that the radiosity is the black body emission.

The cylinder lining is adiabatic, (2 marks) so the heat flow into node 3 is equal to the heat flow out of node 3:

$$Energy\ balance\ means \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0. Note\ R_{23} = R_{13}, (2\ marks)$$

$$So\ 2J_3 = J_1 + J_2 \rightarrow J_3 = \frac{(4.735 + 1.055)}{2} \times 10^5 = 2.8953 \times 10^5\ K/W \quad (1\ marks)$$

Simply take the 4th root (or the square root of the square root) of the radiosity J_2 to get the answer.

$$J_3 = \sigma T_3^4 \rightarrow T_3 = \left(\frac{J_3}{\sigma}\right)^{\frac{1}{4}} = 1503\ K\ (21 marks)$$

(f) Briefly explain how this analysis could be extended to make it more realistic. [3]

Conduction through crown and head. Conduction through liner. Etc. Gray body. (1 marks for added item and 2 mark for explanation)

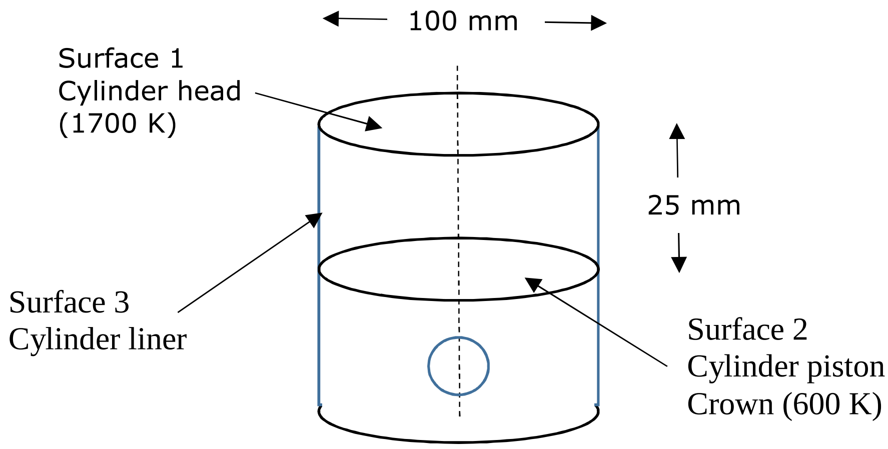
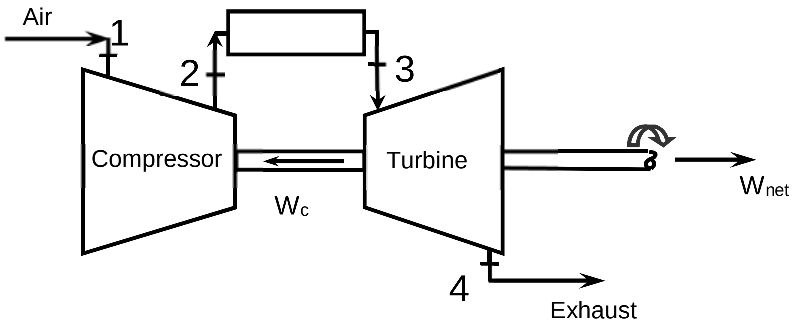

100 mm
Surface 1
Cylinder head
(1700 K)
25 mm
Surface 3
Cylinder liner
Surface 2
Cylinder piston
Crown (600 K)

Fig BQ1

18. A gas turbine operates in an environment in ambient conditions -2°C and 985 mbar. The combustion chamber is at a pressure of 13.9 bar and the turbine inlet temperature is 1380°C. The isentropic efficiency of compressor and turbine are both 88% and adiabatic index (or ratio of specific heats γ) is 1.4 in the compressor and 1.297 in the turbine.

(a) Calculate the compressor and turbine outlet temperatures.



At air inlet p_1= 0.985 bar, T_1 = -2°C (271 K) and p_2 = 13.9 bar
For compressor, η_c = 0.88 and γ = 1.4

At the turbine inlet p_3 = 13.9 bar, T_3 = 1380°C (1653 K) and p_4 = 0.986 bar
For turbine η_c = 0.88 and γ = 1.297

For an isentropic compressor

$$T_2' = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$ [3 marks]

$$T_2' = 271\left(\frac{13.96}{0.986}\right)^{\frac{1.4-1}{1.4}} = 577.8\text{K}$$

Equation for isentropic efficiency for a compressor is:

$$\eta_c = \frac{W_c'}{W_c} = \frac{\Delta h_c'}{\Delta h_c} = \frac{(T_2'-T_1)}{(T_2-T_1)}$$

So T_2 = 271 + (577.8 – 271)/0.88 = 620 K (347°C) [3 marks]

For in isentropic turbine:

$$T_4' = T_3\left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4' = 1653\left(\frac{0.986}{13.96}\right)^{\frac{1.297-1}{1.297}} = 900.7\text{K}$$ [2 marks]

Equation for isentropic efficiency for turbine is:

$$\eta_c = \frac{W_t}{W_t'} = \frac{\Delta h_t}{\Delta h_t'} = \frac{(T_3-T_4)}{(T_3-T_4')}$$

So T_4 = 1653 - 0.88(1653 – 901) = 991K (718°C) [2 marks]

(b) Determine the cycle efficiency (assuming an ideal air cycle).

Cycle efficiency $\eta = W_{net}/Q_{in}$ [2 marks]

For an ideal air cycle, in which it is assumed that the working fluid is air in all components, W_{net} = $W_{net} = \dot{m}c_p(T_3 - T_4) - \dot{m}c_p(T_2 - T_1)$
And $Q_{in} = \dot{m}c_p(T_3 - T_2)$
Since mass flow and c_p terms cancel out:

Cycle efficiency $\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_4)}$ [3 marks]

$\eta = \frac{(1380-718)-(347-(-2))}{(1380-347)}$ = 0.303 (30.3%) [3 marks]

(c) A turbine stage is to be designed with a flow coefficient of 0.6, a reaction of 50% and a stage loading of 3.5. Find the entry and exit relative flow angles and sketch the velocity triangles; calculate the axial velocity and given that the speed is 3000 rpm and the mean radius is 0.25 m, calculate the mass flow rate for a blade height of 0.02 m and air at 1 bar and 300 K.

Assume that the formulae for a repeating stage turbine can be used, in which it is assumed that c_x is constant, and we can use the simplified formulae, as given in the formula sheet as follows:
Flow coefficient: $\phi = \frac{c_x}{U} = 0.6$
Reaction: $R = 0.5 = \frac{\phi}{2}(\tan\beta_3 - \tan\beta_2)$
Stage loading coefficient: $\psi = 3.5 = \phi(\tan\beta_3 + \tan\beta_2)$
Putting in the numbers given and rearranging produces β_3 = 75.1° and β_2 = 64.4° [5 marks]

$$0.5 = \frac{0.6}{2}(\tan\beta_3 - \tan\beta_2)$$
$$3.5 = 0.6(\tan\beta_3 + \tan\beta_2)$$
$$\frac{2\times 0.5}{0.6} + \tan\beta_2 = \tan\beta_3$$
$$3.5 = 0.6\left(\frac{2\times 0.5}{0.6} + 2\tan\beta_2\right) \rightarrow \tan\beta_2 = \frac{\frac{3.5}{0.6} - \frac{2\times 0.5}{0.6}}{2} = 2.08 \rightarrow \beta_2 = 64.4°$$
$$\frac{2\times 0.5}{0.6} + 2.08 = \tan\beta_3 = 3.74 \rightarrow \beta_3 = 75.1°$$

Blade speed is required to work out the axial velocity c_x, which is:
$$U = 2\pi\frac{3000}{60}\times 0.25 = 78.5\ m/s$$
Therefore c_x = 0.6×78.5 = 54.9 m/s [3 marks]

Gas density is from [1 mark]:
$$\rho = \frac{p}{RT} = \frac{98500}{287\times 271} = 1.27\ kg/m^3$$
Annulus area is [1 mark]:
$$A_n = 2\pi rh = 2\pi \times 0.25 \times 0.02 = 0.031\ m^2$$
Flow rate is [2 mark]:
$$\dot{m} = \rho A_n c_x = 1.27 \times 0.031 \times 54.9 = 2.161\ kg/s$$